Fuzzy Systems and Soft Computing
ISSN : 1819-4362

ENHANCING FUZZY EOQ IN JUDICIAL FRAMEWORK USING MACHINE LEARNING WITH WEKA

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Abstract

Budgets for the legal industry in India are frequently established based on continual costs, such as pay rates and operating expenditures, without regard for capacity building or intended outcomes. Despite the fact that complying with the law necessitates significant budgetary expenditures, the specifics of this relationship have not been thoroughly examined. Previous researchers have investigated the effects of legal investing on efficiency and execution, but none have used machine learning techniques. In this study, we use machine learning methodologies to assess the judicial framework's efficiency in terms of investment and other criteria. The clustering process allows us to rank Indian courts based on their efficiency, whereas the EOQ distinguishes the household-level features relevant to each cluster. The findings highlight that simply increasing open-budget investing is insufficient to improve the performance of the equity framework. Extra-authoritarian actions are essential for meeting the demands of the Indian equity framework and accelerating the recovery rate. To solve these issues, we apply the Kuhn-Tucker Strategy for Nonlinear Programming to identify optimal arrangements for legal use and efficiency. Furthermore, our proposed model combines the use of Pentagonal fuzzy numbers to determine the minimum cost, while applying the Signed distance method for defuzzification, analysing use, and using the Weka Software to approve our inventory model, employing a CSV record containing a standard arrangement amount and adding up to fetch. Visualization is done by SPSS. These emerge as crucial, speculative, and authoritative commitments at this time.

Keywords:
Productivity, Judiciary, Economic Order Quantity, Optimal Total Cost, Pentagonal Fuzzy Number (PFN), Kuhn-Tucker Method, Graded Mean Integration Representation Method (GMI), Signed Distance Method (SM), Weka.

Introduction:

Equity postponed becomes equity denied. Productive judicial systems are critical for maintaining the legitimacy of a country's legislative structure and avoiding losing trust in the political structure as a whole, that can have significant financial consequences [1]. The law is a major organisation for organising society since it helps to construct a courteous society and uphold the rule of law [2].

It is clear that the successful organisation of the equity framework is critical to the support and stability of any legal system [3]. Furthermore, there is a rising recognition that an equitable framework capable of properly resolving issues may be a necessary prerequisite for nations' financial growth [4].

Despite its relevance, the law confronts several obstacles in dealing with the large number of cases it receives each year. One of these issues is to increase productivity and reduce processing time. The ineffective execution of the law may be a global issue that has been extensively investigated by various creators and is not unique to the Indian context [5].

One approach to legal execution is to consider its determinants, which frequently examine the legality of a certain nation as a whole [6].

This approach takes into account both internal components such as the framework of the law, the quantity of legal units, the number of officials and judges, and the resources that were contributed, and external components such as training files, economic markers, topographical elimination of legal units, social helplessness of the locale, social angles in relation to citizens, and other features [7].

In a World Bank report [8], a quantitative study examined the variables influencing legal production in ten developing countries across three continents. They conducted legal research to determine how various factors, such as budgetary constraints and judicial authority, influence the length of procedures. The data was divided into three main categories: procedural, administrative, and organisational variables [9,10,11].

The workload is a major issue in the judiciary's work because it is widely known that Indian judges are under unusual strain as a result of the high volume of cases, which can reach millions in a few regions of the country. The National Legal Board's (CNJ) reports confirm the reality of this situation [12,13].

In any case, achieving this goal is not without its hurdles. These include judges' excessive workloads, the necessity for legal modifications that go beyond minor adjustments in procedural legislation, and the effect of factors like engagement and the quantity of counsellors on judiciary efficiency [14,15].

The purpose of this article is to investigate legal efficiency based on usage and other criteria using machine learning techniques. The goal is to determine whether assets are allocated and used effectively, both in terms of goods and human resources, and to investigate whether productivity corresponds to how these assets are divided.

The impact of craftsmanship on legal efficiency

It demonstrates that open-segment organisations have grown to be a demanding challenge for open-minded founders, directors, and respectful hires [17]. This condition makes administration more difficult in developing countries and within the financial sector, as is the case in India. In the public sector, effectiveness, viability, and the ability to meet residents' needs are essential.

It is possible to observe that getting closer to the concept of execution entails addressing specific challenges that analysts must examine. These challenges include: (1) having an effective hypothetical foundation on the nature of execution, that is, a hypothesis that establishes which measures are appropriate in a given research setting; and (2) relying on a reliable hypothesis about the nature of the measures, that is, the presence of writing that describes which measures should be combined and which strategy would be best suited to construct such measures [18].

The variables that influence the productiveness or wastefulness of the judiciary's performance have been thoroughly explored in the writing, particularly the precursors of efficiency. One factor that influences legal execution is the total amount of cases per officer, also known as a legal request. A few creators have suggested that an increase in requests leads to an increase in the judiciary's execution, but there is a limit to this relationship, and execution begins to decline as the number of cases per judge rises [19].

The connection between judicial execution and judicial request is not obvious, and a quadratic graph provides far more information about how the legal system works [20]. An rise in requests can initially boost performance since judges take less time to analyse a case to avoid case accumulation, but this link is not fundamentally true. For example, despite a rise in requests made by the National Board of Equity (CNJ), the legal effectiveness has remained stable for a long time [21].

To assess the effectiveness of the legal system, a few elements have been regarded significant, such as the number of legal counsellors, server infrastructure, workload, and outsourced representatives, while others, such as the amount of conciliators, have been eliminated [22,23].

A few variables, such as personnel proportion and task complexity, have an impact on legal efficiency. Higher courts are particularly influenced by staff ratios, which can extend the time required to process claims [24,25,26]. In any event, inconsistencies in information collecting can impede the study of efficiency in specific sectors of the law that perform admirably in some instances but not in others. The allocation of human resources based on workload has been identified as a concern due to the requirement for a relationship between the number of officers and the demand for courts, particularly in outlying districts. These findings complement studies that suggests that proficiency can be increased without increasing the number of assets used, particularly human asset inputs. Creating specialised courts is an efficient technique for increasing speed without increasing costs. Since it focusses legal units within judgements on requests for related objects, resulting in a type of division of labour or specialisation of capacities.

Preliminaries

1.1: Fuzzy Set

Let U be a nonempty set. Then a fuzzy set G in U (ie., a fuzzy subset A of U) is characterized by a function of the form μ_G : U $\rightarrow$ [0,1]. Such a function μ_G is called the membership function and for each g $\in$ G, $\mu_{\tilde{G}}(x)$ is the degree of membership of u in the fuzzy set G.

1.2 Pentagonal Fuzzy Number

A Pentagonal Fuzzy Number can be represented as $\tilde{e}_p = (e_1, e_2, e_3, e_4, e_5)$, where e_1, e_2, e_3, e_4, e_5 are real numbers. These five real numbers (coordinates) shape the pentagon of the fuzzy set.

The Membership function $F\tilde{A}(x)$ is defined as follows:

$$F\tilde{A}(x) = \begin{cases} 0, & x < e_1 \\ \frac{x - e_1}{e_2 - e_1}, & e_1 \le x \le e_2 \\ \frac{x - e_2}{e_3 - e_2}, & e_2 \le x \le e_3 \\ 1, & x = e_3 \\ \frac{e_4 - x}{e_4 - e_3}, & e_3 \le x \le e_4 \\ \frac{e_5 - x}{e_2 - e_1}, & e_4 \le x \le e_5 \\ 0, & x > e_5, \end{cases} \quad \text{............(1)}$$

These map real numbers to a degree of membership between 0 and 1, defining how much each number belongs to the fuzzy set.

1.3 The Fuzzy Arithmetical Operations Under Function Principle

Chen introduced the Function Principle. The function principle is based on the fuzzy arithmetic operations of pentagonal fuzzy integers. The fuzzy arithmetic principle for Pentagonal Fuzzy Numbers, which is a generalisation of regular arithmetic applicable to PFNs, is next described. This includes operations such as addition, subtraction, multiplication, and division. certain equations are important because they formally define how certain operations are performed on PFNs. The Function Principle defines the following fuzzy arithmetic procedures:

Suppose $\tilde{Q} = (q_1, q_2, q_3, q_4, q_5)$ & $\tilde{R} = (r_1, r_2, r_3, r_4, r_5)$ are two pentagonal fuzzy numbers.

1) $\tilde{Q} \oplus \tilde{R} = (q_1 + r_1, q_2 + r_2, q_3 + r_3, q_4 + r_4, q_5 + r_5)$

2) $\tilde{Q} \otimes \tilde{R} = (q_1 r_1, q_2 r_2, q_3 r_3, q_4 r_4, q_5 r_5)$

3) $\tilde{Q} \ominus \tilde{R} = (q_1 - r_5, q_2 - r_4, q_3 - r_3, q_4 - r_2, q_5 - r_1\}$

4) $\frac{\tilde{Q}}{\tilde{R}} = \left(\frac{q_1}{r_5}, \frac{q_2}{r_4}, \frac{q_3}{r_3}, \frac{q_4}{r_2}, \frac{q_5}{r_1} \right)$

5) Let $\propto \epsilon R$ then:

(i) $\alpha \ge 0, \alpha \otimes \tilde{X} = (\alpha x_1, \alpha x_2, \alpha x_3, \alpha x_4, \alpha x_5)$

(ii) $\alpha \le 0, \alpha \otimes \tilde{X} = (\alpha x_5, \alpha x_4, \alpha x_3, \alpha x_2, \alpha x_1)$

1.4 Graded Mean Integration Representation Method (GMIRM)

Chen and Hsieh [16] developed the graded mean integration representation approach, which is based on the integral value of a generalised fuzzy number's graded mean h-level for defuzzification. In this paper, we have used pentagonal fuzzy numbers as fuzzy parameters for the production inventory model. Let $\tilde{I}$ be a pentagonal fuzzy number and denoted as $\tilde{I} = (I_1, I_2, I_3, I_4, I_5)$. Then we can get the graded mean integration representation of $\tilde{I}$ as

$$P(\tilde{I}) = \frac{\int_0^w \frac{h}{2}\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh}{\int_0^w h\,dh} = \frac{I_1 + 2I_2 + 2I_3 + 2I_4 + I_5}{8}$$

Fuzzy Inventory Economic Order Quantity Model

Understanding the suggested model requires familiarity with the symbols used in the manuscript. Table 2 contains a list and descriptions of these symbols.

Assumptions

It is critical to explicitly express the underlying assumptions of every mathematical model constructed. Certain assumptions are made to ensure that the model functions properly and appropriately depicts the phenomenon under investigation. The recommended model is based on the following assumptions:

The combined inventory model for a specific expense considers only judicial productivity and expenditure in terms of material and human resources. (i) Demand for expenses remains constant over time (ii) Shortages are not allowed. (iii) The lead time, denoted as L, is made up of separate components (iv) The advisor's willingness to accept payment delays from law students saves them money on intern fees each year and (v)Time is supposed to be endless.

Table 2. Notation Applied

Symbols	Description
T	Total Spending
J	Published Judgements
L	Field Lawsuits
H	Instruction Hearings
P	Preliminary Hearings
S	Decisions
R	Orders
U	Sentences
C	Second Degree Commissioned Positions
Q	Intern Expenses
A	Total Personnel Asset Expenses
F	First Degree Personnel Asset Expenses
V	Second Degree Personnel Asset Expenses
W	Total Workforce
M	Magistrate Productivity Index
N	Server Productivity Index

Formulation of the Inventory Economic Order Quantity Model

The EOQ Model, also referred to as the Wilson Model, seeks to balance acquisition and maintenance expenses. The EOQ Model is based on several assumptions, such as the Magistrate Productivity Index, Server Productivity Index, Decisions, Orders, and Intern Expenses. The EOQ Model for the Integrated Inventory Model yielded the following total profit (TC) for Intern Expenses (Q):

$$TC = \frac{QJ}{PS} + \frac{M + NV}{Q} + \frac{QSR}{UC} + \frac{AFW}{Q} + \frac{AFWR}{Q} + \frac{SQC}{HL}$$

Differentiating partially equation with respect to Q, and:

Put $\frac{\partial TC}{\partial Q} = 0$, we get $Q = \sqrt{\frac{(M + NV) + AFW + AFWR}{\frac{J}{PS} + \frac{SR}{UC} + \frac{SC}{HL}}}$

An Integrated Inventory Judicial Model for Crisp Production Quantity

On the whole, in this paper, we use the following variables in order to simplify the use of integrated inventory models. $\tilde{J}, \tilde{V}, \tilde{R}$ are the fuzzy parameters. We establish an integrated inventory model with fuzzy parameters for crisp production quantity $D\tilde{T}C(Q)$ as follows:

The annual integrated total inventory cost is,

$$D\tilde{T}C(Q) = \left[\frac{Q_1 J_1}{PS} + \frac{M+NV_1}{Q} + \frac{Q_1 SR_1}{UC} + \frac{AFW}{Q} + \frac{AFWR_1}{Q} + \frac{SQ_1 C}{HL}\right],$$
$$\left[\frac{Q_2 J_2}{PS} + \frac{M+NV_2}{Q} + \frac{Q_2 SR_2}{UC} + \frac{AFW}{Q} + \frac{AFWR_2}{Q} + \frac{SQ_2 C}{HL}\right],$$
$$\left[\frac{Q_3 J_3}{PS} + \frac{M+NV_3}{Q} + \frac{Q_3 SR_3}{UC} + \frac{AFW}{Q} + \frac{AFWR_3}{Q} + \frac{SQ_3 C}{HL}\right],$$
$$\left[\frac{Q_4 J_4}{PS} + \frac{M+NV_4}{Q} + \frac{Q_4 SR_4}{UC} + \frac{AFW}{Q} + \frac{AFWR_4}{Q} + \frac{SQ_4 C}{HL}\right],$$
$$\left[\frac{Q_5 J_5}{PS} + \frac{M+NV_5}{Q} + \frac{Q_5 SR_5}{UC} + \frac{AFW}{Q} + \frac{AFWR_5}{Q} + \frac{SQ_5 C}{HL}\right]$$

Suppose, $\tilde{J} = (J_1, J_2, J_3, J_4)$, $\tilde{V} = (V_1, V_2, V_3, V_4)$, $\tilde{R} = (R_1, R_2, R_3, R_4)$ are non-negative trapezoidal fuzzy numbers. Then we solve for the optimal production quantity in the following steps. Next, we defuzzify the fuzzy total production inventory for the expected profit using the formula (a).

$$P(D\tilde{T}C(Q)) = \frac{1}{8}\left\{\begin{array}{l} \left[\frac{Q_1 J_1}{PS} + \frac{M+NV_1}{Q} + \frac{Q_1 SR_1}{UC} + \frac{AFW}{Q} + \frac{AFWR_1}{Q} + \frac{SQ_1 C}{HL}\right] + \\ 2\left[\frac{Q_2 J_2}{PS} + \frac{M+NV_2}{Q} + \frac{Q_2 SR_2}{UC} + \frac{AFW}{Q} + \frac{AFWR_2}{Q} + \frac{SQ_2 C}{HL}\right] + \\ 2\left[\frac{Q_3 J_3}{PS} + \frac{M+NV_3}{Q} + \frac{Q_3 SR_3}{UC} + \frac{AFW}{Q} + \frac{AFWR_3}{Q} + \frac{SQ_3 C}{HL}\right] + \\ 2\left[\frac{Q_4 J_4}{PS} + \frac{M+NV_4}{Q} + \frac{Q_4 SR_4}{UC} + \frac{AFW}{Q} + \frac{AFWR_4}{Q} + \frac{SQ_4 C}{HL}\right] + \\ \left[\frac{Q_5 J_5}{PS} + \frac{M+NV_5}{Q} + \frac{Q_5 SR_5}{UC} + \frac{AFW}{Q} + \frac{AFWR_5}{Q} + \frac{SQ_5 C}{HL}\right] \end{array}\right\}$$

To find the minimization of $P(D\tilde{T}C(Q))$, we have to differentiate partially with respect to Q and equate it to zero. We can get it as follows:

$$Q^* = \sqrt{\frac{\begin{array}{c}[(M+NV_1) + 2((M+NV_2) + (M+NV_3) + (M+NV_4)) + (M+NV_5)] + \\ 8AFW + [AFWR_1 + 2((AFWR_2) + (AFWR_3) + (AFWR_4)) + AFWR_5]\end{array}}{\left[\frac{J_1}{PS} + 2\left(\frac{J_2}{PS} + \frac{J_3}{PS} + \frac{J_4}{PS}\right) + \frac{J_5}{PS}\right] + \left[\frac{SR_1}{UC} + 2\left(\frac{SR_2}{UC} + \frac{SR_3}{UC} + \frac{SR_4}{UC}\right) + \frac{SR_5}{UC}\right] + 8\left[\frac{SC}{HL}\right]}}$$

An Integrated Inventory Judicial Model for Fuzzy Production Quantity DC (Q)

Suppose, $\tilde{J} = (J_1, J_2, J_3, J_4)$, $\tilde{V} = (V_1, V_2, V_3, V_4)$, $\tilde{R} = (R_1, R_2, R_3, R_4)$ and by applying the Graded Mean Integration Representation Method by using formula (a), we can get it as follows:

$$P(D\tilde{T}C(Q))=\frac{1}{8}\left\{\begin{array}{l}\left[\frac{Q_1 J_1}{PS}+\frac{M+NV_1}{Q_5}+\frac{Q_1 SR_1}{UC}+\frac{AFW}{Q_5}+\frac{AFWR_1}{Q_5}+\frac{SQ_1 C}{HL}\right]+\\ 2\left[\frac{Q_2 J_2}{PS}+\frac{M+NV_2}{Q_4}+\frac{Q_2 SR_2}{UC}+\frac{AFW}{Q_4}+\frac{AFWR_2}{Q_4}+\frac{SQ_2 C}{HL}\right]+\\ 2\left[\frac{Q_3 J_3}{PS}+\frac{M+NV_3}{Q_3}+\frac{Q_3 SR_3}{UC}+\frac{AFW}{Q_3}+\frac{AFWR_3}{Q_3}+\frac{SQ_3 C}{HL}\right]+\\ 2\left[\frac{Q_4 J_4}{PS}+\frac{M+NV_4}{Q_2}+\frac{Q_4 SR_4}{UC}+\frac{AFW}{Q_2}+\frac{AFWR_4}{Q_2}+\frac{SQ_4 C}{HL}\right]+\\ \left[\frac{Q_5 J_5}{PS}+\frac{M+NV_5}{Q_1}+\frac{Q_5 SR_5}{UC}+\frac{AFW}{Q_1}+\frac{AFWR_5}{Q_1}+\frac{SQ_5 C}{HL}\right]\end{array}\right\} \quad ....(1)$$

Now differentiate partially (1) with respect to Q_1, Q_2, Q_3 and Q_4 and equate it to zero. We can get it as follows:

$$Q^*=\sqrt{\frac{\begin{array}{c}[(M+NV_1)+2((M+NV_2)+(M+NV_3)+(M+NV_4))+(M+NV_5)]+\\ 8AFW+[AFWR_1+2((AFWR_2)+(AFWR_3)+(AFWR_4))+AFWR_5]\end{array}}{\left[\frac{J_1}{PS}+2\left(\frac{J_2}{PS}+\frac{J_3}{PS}+\frac{J_4}{PS}\right)+\frac{J_5}{PS}\right]+\left[\frac{SR_1}{UC}+2\left(\frac{SR_2}{UC}+\frac{SR_3}{UC}+\frac{SR_4}{UC}\right)+\frac{SR_5}{UC}\right]+8\left[\frac{SC}{HL}\right]}}$$

Solution Methods and Kuhn-Tucker Model Formulation and its Condition

Let us consider a problem formulated as $\min\, y=f(x)$, subject to constraints $\varepsilon_i(x)\geq 0$, for i = 1,…….,m. If there are any non-negativity constraints, x ≥ 0, they are included in the m constraints. Here x represents the vector of decision variables and $f(x)$ is the objective function to be minimized. The constraints $\varepsilon_i(x)\geq 0$ are the restrictions imposed on the decision variables, with m denoting the number of constraints. The constraints include non-negativity restrictions (X > 0), where X represents the feasible set of decision variables.

To transform the inequality constraints into equations, non-negative surplus variables can be introduced. Let $T_i^{\ 2}$ represents the surplus quantity added to the ith constraint $\varepsilon_i(x)\geq 0$. Thus, we have $\psi=(\psi_1,\psi_2,........,\psi_m)$, $g(x)=(\varepsilon_1(x),\varepsilon_2(x),.....,\varepsilon_m(x))$ and $T^2 = (T_1^2, T_2^2, ..., T_m^2)$.

where ψ represents the vector of Lagrange multipliers associated with the constraints, $g(x)$ is the vector of constraints, and T^2 is the vector of surplus variables.

To satisfy the Kuhn-Tucker criteria, it needs the solution vectors X and ψ of the minimized problem to be considered a stationary point, which must satisfy the following conditions, which can be summarized as follows:

$$\left\{\begin{array}{l}\mu\leq 0\\ \nabla f(x)-\mu\nabla g(x)=0,\\ \mu_i g_i(x)=0, i=1,2,......m\\ g_i(x)\geq 0, i=1,2,.......m\end{array}\right\}$$

where ψ represents the vector of Lagrange multipliers, $\nabla f(x)$ is the gradient of the objective function, and $\nabla\varepsilon(x)$ is the gradient of the constraints.

An Integrated Inventory Judicial Model for Fuzzy Production Quantity DC (Q) by using Kuhn-Tucker Conditions

In this case, we introduce the fuzzy inventory EOQ models into the fuzzy order quantity $\tilde{Q}$ as a pentagonal fuzzy number $\tilde{Q}=$ (Q_1,Q_2,Q_3,Q_4,Q_5) with fuzzy production quantity as $0<Q_1\leq Q_2\leq Q_3\leq Q_4\leq Q_5$. Suppose, $\tilde{J}=(J_1,J_2,J_3,J_4)$, $\tilde{V}=(V_1,V_2,V_3,V_4)$, $\tilde{R}=(R_1,R_2,R_3,R_4)$ and next, we have to defuzzify the fuzzy average individual cost for retailer by applying the Graded Mean

Integration Representation Method of $P(D\tilde{T}C(Q))$ by using formula (a), we can get the result as follows:

$$P(D\tilde{T}C(Q)) = \frac{1}{8}\left\{\begin{array}{l} \left[\frac{Q_1 J_1}{PS} + \frac{M+NV_1}{Q_5} + \frac{Q_1 SR_1}{UC} + \frac{AFW}{Q_5} + \frac{AFWR_1}{Q_5} + \frac{SQ_1 C}{HL}\right] + \\ 2\left[\frac{Q_2 J_2}{PS} + \frac{M+NV_2}{Q_4} + \frac{Q_2 SR_2}{UC} + \frac{AFW}{Q_4} + \frac{AFWR_2}{Q_4} + \frac{SQ_2 C}{HL}\right] + \\ 2\left[\frac{Q_3 J_3}{PS} + \frac{M+NV_3}{Q_3} + \frac{Q_3 SR_3}{UC} + \frac{AFW}{Q_3} + \frac{AFWR_3}{Q_3} + \frac{SQ_3 C}{HL}\right] + \\ 2\left[\frac{Q_4 J_4}{PS} + \frac{M+NV_4}{Q_2} + \frac{Q_4 SR_4}{UC} + \frac{AFW}{Q_2} + \frac{AFWR_4}{Q_2} + \frac{SQ_4 C}{HL}\right] + \\ \left[\frac{Q_5 J_5}{PS} + \frac{M+NV_5}{Q_1} + \frac{Q_5 SR_5}{UC} + \frac{AFW}{Q_1} + \frac{AFWR_5}{Q_1} + \frac{SQ_5 C}{HL}\right] \end{array}\right\}$$

with $0 < Q_1 \le Q_2 \le Q_3 \le Q_4 \le Q_5$.

If we replace the inequality conditions $0 < Q_1 \le Q_2 \le Q_3 \le Q_4 \le Q_5$ into the following inequality condition $Q_2 - Q_1 \ge 0$, $Q_3 - Q_2 \ge 0$, $Q_4 - Q_3 \ge 0$, $Q_5 - Q_4 \ge 0$, $Q_1 > 0$ then the meaning of the above formula remains unchanged.

Next, we have use the Kuhn-Tucker condition to find the solution of Q_1, Q_2, Q_3 and Q_4 and to minimize $P(J\tilde{T}C(Q))$ and then subject it to $Q_2 - Q_1 \ge 0$, $Q_3 - Q_2 \ge 0$, $Q_4 - Q_3 \ge 0$, $Q_5 - Q_4 \ge 0$, $Q_1 > 0$. The Kuhn-Tucker conditions are $\psi \le 0$.

$$\nabla f\ (P(J\tilde{T}C\,(Q)) - \mu\nabla g\,(Q) = 0$$

$$\mu_i\, g_i\ (Q) = 0$$

$$g_i\ (Q) = 0$$

These conditions simplify to the following $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \le 0$ and

$$\nabla f\ (P(J\tilde{T}C\,(Q)) - \mu\nabla g\,(Q) = 0$$

Therefore,

$$P(D\tilde{T}C(Q)) = \frac{1}{8}\left\{\begin{array}{l} \left[\frac{Q_1 J_1}{PS} + \frac{M+NV_1}{Q_5} + \frac{Q_1 SR_1}{UC} + \frac{AFW}{Q_5} + \frac{AFWR_1}{Q_5} + \frac{SQ_1 C}{HL}\right] + \\ 2\left[\frac{Q_2 J_2}{PS} + \frac{M+NV_2}{Q_4} + \frac{Q_2 SR_2}{UC} + \frac{AFW}{Q_4} + \frac{AFWR_2}{Q_4} + \frac{SQ_2 C}{HL}\right] + \\ 2\left[\frac{Q_3 J_3}{PS} + \frac{M+NV_3}{Q_3} + \frac{Q_3 SR_3}{UC} + \frac{AFW}{Q_3} + \frac{AFWR_3}{Q_3} + \frac{SQ_3 C}{HL}\right] + \\ 2\left[\frac{Q_4 J_4}{PS} + \frac{M+NV_4}{Q_2} + \frac{Q_4 SR_4}{UC} + \frac{AFW}{Q_2} + \frac{AFWR_4}{Q_2} + \frac{SQ_4 C}{HL}\right] + \\ \left[\frac{Q_5 J_5}{PS} + \frac{M+NV_5}{Q_1} + \frac{Q_5 SR_5}{UC} + \frac{AFW}{Q_1} + \frac{AFWR_5}{Q_1} + \frac{SQ_5 C}{HL}\right] \end{array}\right\}$$

$$- \mu_1 (Q_2-Q_1) - \mu_2 (Q_3-Q_2) - \mu_3 (Q_4-Q_3) - \mu_4 (Q_5-Q_4) - \mu_5 (Q_1) = 0$$

which implies

that

$$\frac{1}{8}\left[\frac{Q_1 J_1}{PS} + \frac{M+NV_1}{Q_5} + \frac{Q_1 SR_1}{UC} + \frac{AFW}{Q_5} + \frac{AFWR_1}{Q_5} + \frac{SQ_1 C}{HL}\right] + \mu_1 - \mu_5 = 0$$

$$\frac{2}{8}\left[\frac{Q_2 J_1}{PS}+\frac{M+NV_2}{Q_4}+\frac{Q_2 SR_2}{UC}+\frac{AFW}{Q_4}+\frac{AFWR_2}{Q_4}+\frac{SQ_2 C}{HL}\right]+\mu_1-\mu_2=0$$

$$\frac{2}{8}\left[\frac{Q_3 J_3}{PS}+\frac{M+NV_3}{Q_3}+\frac{Q_3 SR_3}{UC}+\frac{AFW}{Q_3}+\frac{AFWR_3}{Q_3}+\frac{SQ_3 C}{HL}\right]-\mu_2+\mu_3=0$$

$$\frac{2}{8}\left[\frac{Q_4 J_4}{PS}+\frac{M+NV_4}{Q_2}+\frac{Q_4 SR_4}{UC}+\frac{AFW}{Q_2}+\frac{AFWR_4}{Q_2}+\frac{SQ_4 C}{HL}\right]-\mu_3+\mu_4=0$$

$$\frac{1}{8}\left[\frac{Q_5 J_5}{PS}+\frac{M+NV_1}{Q}+\frac{Q_1 SR_1}{UC}+\frac{AFW}{Q}+\frac{AFWR_1}{Q}+\frac{SQ_1 C}{HL}\right]-\mu_4=0$$

$\mu_1(Q_2 - Q_1) = 0$ $\mu_2(Q_3 - Q_2) = 0$ $\mu_3(Q_4 - Q_3) = 0$

$\mu_4(Q_5 - Q_4) = 0$ $\mu_5(Q_1) = 0$

$Q_2 - Q_1 \geq 0$ $Q_3 - Q_2 \geq 0$ $Q_4 - Q_3 \geq 0$ $Q_5 - Q_4 \geq 0$ $Q_1 > 0$

Since, $Q_1 > 0$ *and* $\mu_5(Q1) = 0$ *then* $\mu_5 = 0$.

If $\mu_1=\mu_2=\mu_3=\mu_4=\mu_5=0$ then $\mu_5<\mu_4<\mu_3<\mu_2<\mu_1$, it does not satisfy the condition $0 \leq Q_1 \leq Q_2 \leq Q_3 \leq Q_4 \leq Q_5$.

Therefore, $Q_2 = Q_1$, $Q_3 = Q_2$, $Q_4 = Q_3$, $Q_5 = Q_4$.

That is, $Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = Q^*$.

Hence, we find the optimal order quantity Q* by means of $\tilde{Q}^*=(Q_1, Q_2, Q_3, Q_4, Q_5)$

$$\tilde{Q}^*=\sqrt{\frac{[(M+NV_1)+2((M+NV_2)+(M+NV_3)+(M+NV_4))+(M+NV_5)]+8AFW+[AFWR_1+2((AFWR_2)+(AFWR_3)+(AFWR_4))+AFWR_5]}{\left[\frac{J_1}{PS}+2\left(\frac{J_2}{PS}+\frac{J_3}{PS}+\frac{J_4}{PS}\right)+\frac{J_5}{PS}\right]+\left[\frac{SR_1}{UC}+2\left(\frac{SR_2}{UC}+\frac{SR_3}{UC}+\frac{SR_4}{UC}\right)+\frac{SR_5}{UC}\right]+8\left[\frac{SC}{HL}\right]}}$$

We reduce the overall cost of data collection for fuzzy production by employing a fuzzy inventory model solved using the Kuhn-Tucker technique and fuzzified with pentagonal fuzzy numbers.

Utilizing Machine Learning for Simulation

Weka is a collection of machine learning methods that can be applied straight to a dataset. It comprises tools for association rules, the first data processing, regression, classification, clustering, and visualisation. In other words, machine learning allows computers to learn from data without explicit programming.

In our investigation, the values of total cost and Economic Order Quantity (EOQ) for both crisp and fuzzy inventory models were determined using a CSV file with 26475 rows of parameters. Based on the intern expenses, the overall cost was divided into profit and non-profit categories. Profit increases as expenses fall, while profit declines as expenses increase.

=== Stratified cross-validation ===

=== Summary ===

Correctly Classified Instances	26466	99.966 %
Incorrectly Classified Instances	9	0.034 %
Kappa statistic	0.9985	
Mean absolute error	0.0004	
Root mean squared error	0.0171	
Relative absolute error	0.1701 %	
Root relative squared error	5.0014 %	
Total Number of Instances	26475	

=== Detailed Accuracy By Class ===

TP Rate	FP Rate	Precision	Recall	F-Measure	MCC	ROC Area	PRC Area	Class
0.999	0.000	0.998	0.999	0.999	0.999	1.000	1.000	NON PROFIT

	1.000	0.001	1.000	1.000	1.000	0.999	1.000	1.000	PROFIT
Weighted Avg.	1.000	0.001	1.000	1.000	1.000	0.999	1.000	1.000	

=== Confusion Matrix ===

a b <-- classified as

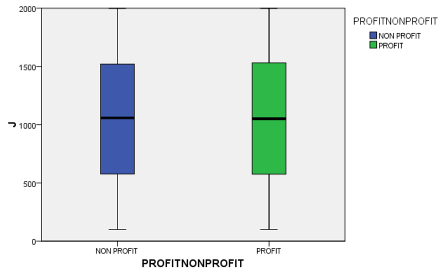
3579 3 | a = NON PROFIT

6 22887 | b = PROFIT

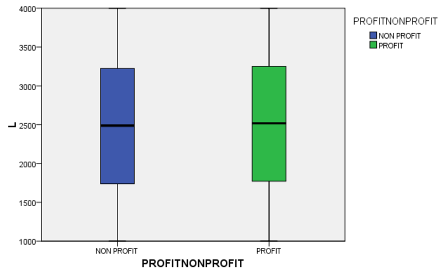
SPSS Software's results visualisation

The IBM SPSS software platform offers a multitude of features, including advanced statistical analysis, text analysis, interaction with big data, a vast library of machine learning algorithms, open source extensibility, and seamless application deployment. Data visualisation is an essential part of data analysis, communication, and exploration. Patterns that are buried in raw numbers are often visible when data is presented visually. Here, the data is visualised using SPSS software. A box plot provides a brief summary of a continuous variable's distribution. They excel at comparing group distributions within your dataset in particular. A box plot provides a lot of information in an understandable way.

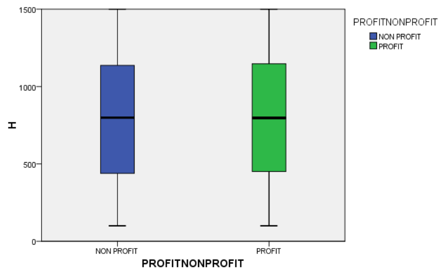
Comparison of various cost factors



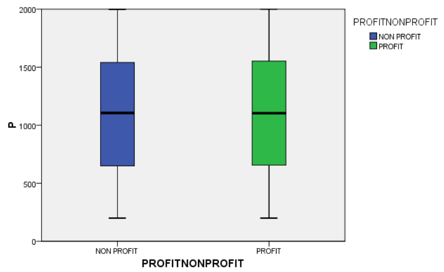
Published Judgments by Profitability



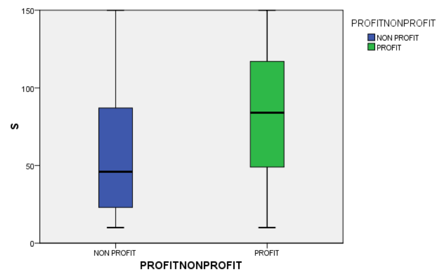
Field Lawsuits by Profitability



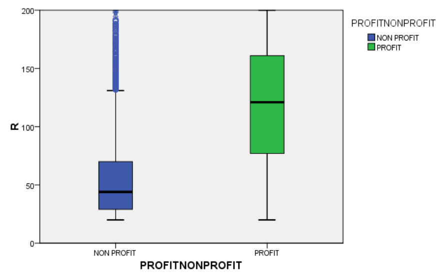
Instruction Hearings by Profitability



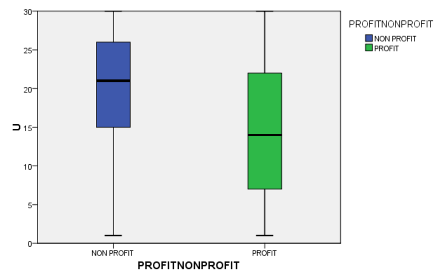
Preliminary Hearings by Profitability



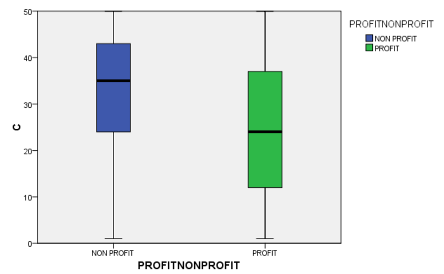
Decisions by Profitability



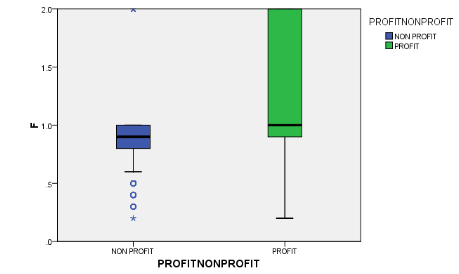
Orders by Profitability



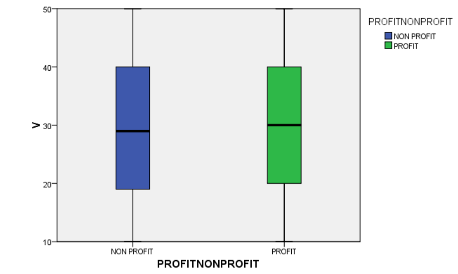
Sentences by Profitability



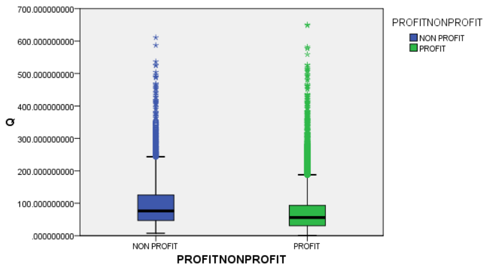
Second degree commisioned positions by Profitability



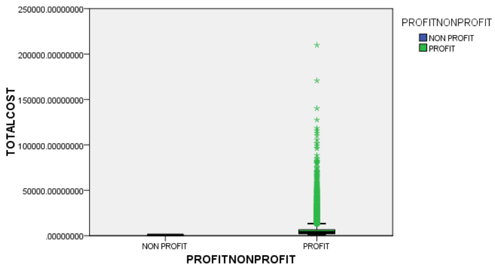
First Degree personnel asset expenses by Profitability



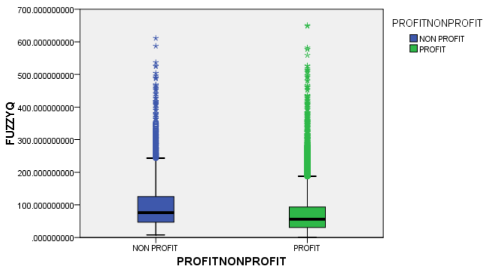
Second Degree personnel asset expenses by Profitability



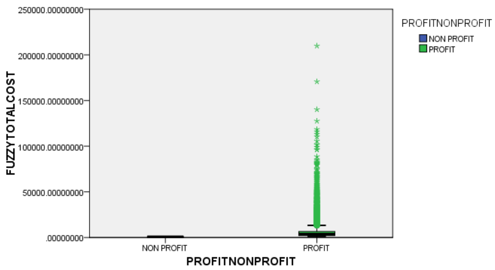
Crisp Intern expenses by Profitability



Crisp Totalcost by Profitability



Fuzzy Intern expenses by Profitability



Fuzzy Totalcost by Profitability

Conclusion

Finally, social transformation is tied to both legal efficiency and the state's jurisdictional function. A right cannot be effective if it does not operate in the context in which it was created. It is futile to create an unlimited number of rules, norms, regulations, and conventions if none of them can influence the real world and control the relationships for which they were intended. In order to accomplish this, the Judiciary must exercise its core function of judging. Judge in a prompt and fair manner.

The purpose of developing an EOQ Model for Reducing Intern Expenses is to reduce total expenses and boost profit while addressing both concrete and ambiguous issues. Defuzzification is achieved using the Graded Mean Integration Representation Method. The Fuzzy EOQ Model can be solved using the Nonlinear Programming Kuhn-Tucker Method. In addition, we employ Weka software to forecast the EOQ Model's values and overall costs. As a result, we have 99.96% accuracy with the provided data. Increasing the value of the EOQ Model, in particular, leads to a rise in total costs.

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